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by Paul S. Segerstrom and Ignat Stepanok

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Keywords: Trade liberalization, heterogeneous firms, quality ladders, endogenous growth.

JEL classification: F12, F13, F43, O31, O41.

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1 Introduction

The issue of which firms export is an important one and has been the topic of many recent papers in the international trade literature. The evidence indicates that even in so-called export sectors, many firms do not export their products. Research has concentrated on two factors to explain the exporting behavior of firms: productivity differences among firms and the presence of fixed costs to entering foreign markets. It has been widely documented that persistent productivity differences exist among firms operating in the same industry and that the more productive and larger firms tend to be the ones that export (see Bernard and Jensen (1999), Aw, Chung and Roberts (2000) and Clerides, Lach and Tybout (1998)). The presence of fixed costs to entering foreign markets has been shown in Bernard and Jensen (2004a) and Roberts and Tybout (1997). Furthermore, Pavcnik (2002), Trefler (2004) and Bernard and Jensen (2004b) have documented that trade liberalization leads to aggregate productivity gains.

In a seminal paper, Melitz (2003) developed the first trade model that is consistent with this empirical evidence. In this model, firms do R&D to develop new product varieties and then learn how costly it is to produce these new products. Once firms have learned what their marginal costs of production are, they decide whether or not to incur the one-time fixed costs of entering the local and foreign markets. The fixed cost of entering the foreign market is assumed to be higher and consequently, only the most productive (lowest marginal cost) firms choose to export their products. When trade liberalization occurs (the variable costs to trade fall), firms earn higher discounted profits from exporting and more firms choose to become exporters. This leads to more competition for all firms in their domestic markets and raises the productivity level required for domestic production. Thus, trade liberalization facilitates the entry of more productive new firms and given the exogenous death rate of old firms, leads to aggregate productivity gains.

In this paper, we present a model of international trade that yields Melitz-type results without the standard Melitz-type assumptions. Instead of assuming that firms do R&D to develop new product varieties, we study a "quality ladders" endogenous growth model where firms do R&D to develop higher quality products. And instead of assuming that firms learn their marginal cost after developing a new product, we assume that there is no uncertainty about the marginal cost of a firm that innovates. Firm heterogeneity emerges naturally in our model because of uncertainty in R&D itself: some firms innovate more quickly than other firms. Thus, at any point in time, different firms produce different quality products and have different profit levels. We show that this quality ladders growth model generates the same empirically supported results about trade liberalization and productivity as Melitz (2003) if it takes time for firms to learn how to export.

The model also has some important properties that differentiate it from Melitz (2003).

First, the model has an endogenously determined firm exit rate that is affected by trade liberalization. This endogeneity comes naturally, since the model has a quality ladders structure. Firms do R&D to develop higher quality products, and when they succeed, they drive the previous quality leaders out of business. Innovation is associated with a process of creative destruction, as was originally emphasized by Schumpeter (1942). We show that trade liberalization (lowering the variable costs to trade) leads to an increase in the exit rate of firms. This result is consistent with the evidence in Pavcnik (2002), where it is reported that a period of trade liberalization in Chile (1979-1986) was accompanied by a "massive" exit rate of firms. Gibson and Harris (1996) have similar findings for New Zealand and Gu, Sawchuk and Rennison (2003) show a significant increase in the exit rate of firms as a result of tariff cuts in Canada during 1989-1996. In Melitz (2003), an exogenous firm exit rate is assumed for firms that have already entered a market (since there is no other reason why firms would choose to go out of business) and consequently trade liberalization has no effect on the exit rate of firms that have already entered a market.

Second, the model implies that exporters charge higher prices on average for their products. There is evidence to support this result: Kugler and Verhoogen (2008) have found that exporters charge higher prices using Colombian data and Hallak and Sivadasan (2009) obtain the same result using Indian and US data. Theoretical models dealing with this empirical regularity either introduce a second source of firm heterogeneity beside productivity (Hallak and Sivadasan 2009) or correlate a firm's marginal cost with product quality (Baldwin and Harrigan 2007, Kugler and Verhoogen 2008). Neither approach is chosen in this paper. Exported products are cheaper in Melitz (2003), which is clearly at odds with the empirical evidence.

Third, the model implies that exporters charge higher markups on average. Recently evidence has emerged to support this result: looking at Slovenian firm-level data for the period between 1994 and 2000, De Loecker and Warzynski (2012) find that exporters charge significantly higher markups on average compared to non-exporting firms. While several models have now been developed to explain why exporting firms charge higher markups, our model is distinctive in that we can explain this empirical regularity while assuming CES consumer preferences (unlike in Melitz and Ottaviano (2008)), linear pricing (unlike in Sugita (2011)) and iceberg trade costs (unlike in Irarrazabal et. al. (2012)). Feenstra and Ma (forthcoming) build a monopolistic competition model with CES preferences and endogenous markups. In our model, there is less product market competition associated with new products and firms that export tend to sell new products, so these firms are able to charge higher markups (and higher prices) than non-exporting firms. All firms charge the same markup in Melitz (2003), which is clearly at odds with the empirical evidence.

Fourth, since some firms learn to become exporters faster than others, the model implies

that at any point in time, there are some relatively large and productive firms that do not export their products. Bernard et. al. (2003) and Hallak and Sivadasan (2009) have documented that many large and productive firms do not export. The model does not generate a threshold productivity level like in Melitz (2003), where all the firms with productivity above the threshold export and all the firms with productivity below the threshold do not export. The Melitz model can explain why many firms do not export but it cannot explain why many large firms do not export.

The rest of this paper is organized as follows: In Section 2, we present our model of international trade without Melitz-type assumptions. We show that this model has a steady-state equilibrium and derive five equilibrium properties (Propositions 1 through 5). In Section 3, we solve the model numerically for plausible parameter values to obtain the effects of trade liberalization on economic growth and welfare. In Section 4, we offer some concluding comments and in the Appendix, we present calculations done to solve our model in more detail.

2 The Model

2.1 Overview

The model presented in this paper is essentially a two country version of the Segerstrom (2007) quality ladders endogenous growth model with the new assumption that it takes time for firms to learn how to export.

There are two symmetric countries, Home and Foreign. In both countries, there is a constant rate of population growth n and the only factor labor is inelastically supplied. Consumers have constant elasticity of substitution (CES) preferences. Workers are employed in a production sector and in an R&D sector. There is a continuum of differentiated products indexed by $\omega \in [0, 1]$. Each product ω has different possible quality levels denoted by j. Higher values of j denote higher quality and j is restricted to taking on integer values. Firms are involved in R&D races to discover the next higher quality product and when a firm succeeds, it replaces the previous incumbent who was selling product ω as a monopolist. When the state-of-the-art quality product is j, the next quality level to be discovered is $j+1$. Over time each product is pushed up its 'quality ladder.' While holding the patent for the state-of-the-art quality of product ω, a firm starts to sell only in its local market. To become an exporter it must invest in learning how to enter the foreign market. Each firm operates until a higher quality version of its product ω is discovered by another firm from its home market. Non-exporters do not have an incentive to improve on their own products. Exporters do not have an incentive under certain parameter conditions that we assume hold. As a result, only followers do innovative R&D. We solve the model for a symmetric steady-state equilibrium.

2.2 Consumers and Workers

The economy has a fixed number of households. They provide labor, for which they earn wages and save by holding assets of firms that engage in R&D. Each household grows at the rate $n > 0$, hence the supply of labor in the economy at time t can be represented by $L_t = L_0 e^{nt}$. Each household is modelled as a dynastic family that maximizes present discounted utility $U \equiv \int_0^\infty e^{-(\rho-n)t} \ln [u_t] \, dt$, where the consumer subjective discount rate ρ satisfies $\rho > n$. The static utility of a representative consumer defined over all products available within a country at time t is

$$u_t \equiv \left[\int_0^1 \left(\sum_j \lambda^j d(j, \omega, t) \right)^\alpha d\omega \right]^{\frac{1}{\alpha}}. \tag{1}$$

This is a quality-augmented Dixit-Stiglitz consumption index, where $d(j, \omega, t)$ denotes the quantity consumed of a product variety ω of quality j at time t, $\lambda > 1$ is the size of each quality improvement and the product differentiation parameter $\alpha \in (0, 1)$ determines the elasticity of substitution between different products $\sigma \equiv \frac{1}{1-\alpha} > 1$. Since λ^j is increasing in j, (1) captures in a simple way the idea that consumers prefer higher quality products.

Utility maximization follows three steps. The first step is to solve the within-variety static optimization problem. Let $p(j, \omega, t)$ be the price of variety ω with quality j at time t. Households allocate their budget within each variety by buying the product with the lowest quality-adjusted price $p(j, \omega, t)/\lambda^j$. To break ties, we assume that when quality-adjusted prices are the same for two products of different quality, each consumer only buys the higher quality product. We will from now on write $p(\omega, t)$ and $j(\omega, t)$ to denote the price and quality level of the product within variety ω with the lowest quality-adjusted price. Demand for all other qualities is zero.

The second step is to find the demand for each product ω given individual consumer expenditure c_t that maximizes individual utility u_t at time t. Solving this problem yields the demand function

$$d(\omega, t) = \frac{q(\omega, t) p(\omega, t)^{-\sigma} c_t}{P_t^{1-\sigma}}, \tag{2}$$

where $d(\omega, t)$ is demand for the product within variety ω with the lowest quality-adjusted price, $q(\omega, t) \equiv \delta^{j(\omega,t)}$ is an alternative measure of product quality, $\delta \equiv \lambda^{\sigma-1} > 1$ and

$$P_t \equiv \left(\int_0^1 q(\omega, t) p(\omega, t)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$

is a quality-adjusted price index.

The third step is to solve for the path of consumer expenditure c_t over time that maximizes discounted utility subject to the relevant intertemporal budget constraint. Solving this intertemporal problem gives the standard Euler equation $\dot{c}_t/c_t = r_t - \rho$, implying the

individual consumer expenditure grows over time only if the interest rate r_t exceeds the subjective discount rate ρ. A higher interest rate induces consumers to save more now and spend more later, resulting in increasing consumer expenditure over time. Since $\dot{c}_t/c_t$ must be constant over time in any steady-state (or balanced growth) equilibrium, the interest rate must be constant over time and from now on, we will refer to the interest rate as r.

A natural measure of productivity at time t is real output $c_t L_t/P_t$ divided by the number of workers L_t, or c_t/P_t. But c_t/P_t equals u_t as Dixit and Stiglitz (1977) have shown. Thus measuring productivity in this model is equivalent to measuring the static utility level of the representative consumer.

2.3 Product Markets

We solve the model for a symmetric steady-state equilibrium where half of all products ω originate from Home and the other half from Foreign. Every product ω will have a version of it sold in both markets. Home originating products will either be exported to Foreign or produced there by Foreign's competitive fringe. We assume that once a better version j of a product originating from Home is discovered, the blueprint of its previous version $j-1$ becomes common knowledge in both Home and Foreign, and can be produced by the competitive fringe in Foreign. Production by the competitive fringe in Foreign continues until the new incumbent in Home learns how to export, starts to sell that product of quality j in Foreign and drives the competitive fringe there with its $j-1$ version out of business. Thus some of the Home originating products having a more advanced version sold in Home and with corresponding assumptions for the Foreign country, some of the Foreign originating products will have a one step higher quality version sold in Foreign.

The production of output is characterized by constant returns to scale. It takes one unit of labor to produce one unit of a good regardless of product quality. The wage rate is normalized to one and firms are price-setters. Each firm produces and sells a unique product ω. Profits of a producer depend on what it sells domestically and what it sells abroad if it exports. We assume iceberg trade costs: an exporter needs to ship $\tau > 1$ units of a good in order for one unit to arrive at the foreign destination. Let $\pi_L(\omega, t)$ and $\pi_E(\omega, t)$ denote profits from local sales and from exporting, respectively, for a firm based at Home. Let $d(\omega, t)L_t$ denote demand for product ω in the Home country. Knowing that lower quality products can be produced by the competitive fringe, the profit-maximizing price that quality leaders can charge at home and abroad is the limit price λ if $\lambda < 1/\alpha$, where $1/\alpha$ is the monopoly price. If $\lambda \geq 1/\alpha$, then innovations are drastic and firms find it optimal to charge the monopoly price $1/\alpha$ at home and (for $\lambda \geq \tau/\alpha$) the monopoly price τ/α abroad. Quality leaders disregard the competitive fringe when the innovation step λ is large enough.

We will assume that innovations are not drastic ($\lambda < 1/\alpha$), which translates into quality

leader firms charging the limit price $p_L = p_E = \lambda$ both at home and abroad. This price does not depend on the quality level of a particular product relative to that of other products. Profits are the difference between price and marginal cost times demand $d(\omega, t)L_t$ for product ω at Home, that is, $\pi_L(\omega, t) = (\lambda - 1)d(\omega, t)L_t$. Let $Q_t \equiv \int_0^1 q(\omega, t)d\omega$ be the average quality of all products sold in Home and $y(t) \equiv Q_t \lambda^{-\sigma} c_t / P_t^{1-\sigma}$ be per capita demand for a product of average quality sold by a leader in Home. Substituting for demand, we can rewrite profits from selling locally as

$$\pi_L(\omega, t) = (\lambda - 1)\frac{q(\omega, t)}{Q_t} y(t)L_t.$$

Profits depend on the quality $q(\omega, t)$ of the product sold. This dependence on the quality of the product comes from the demand function, which is essential for the existence of firm heterogeneity. Different product quality levels result in different profits. In comparison to Melitz (2003) and Haruyama and Zhao (2008) where heterogeneity of profits comes from differing marginal costs, we obtain heterogeneity from the revenue side of profits. If we had assumed a Cobb-Douglas utility function ($\sigma = 1$) which results in unit-elastic demand, we would not have that heterogeneity because profits would not depend on product quality (remember that $\delta \equiv \lambda^{\sigma-1}$ in the definition $q(\omega, t) \equiv \delta^{j(\omega,t)}$).

The marginal cost for selling abroad is $\tau > 1$. Assuming that the limit price firms can charge is higher than the iceberg trade cost ($\lambda > \tau$), we can express profits from exporting as

$$\pi_E(\omega, t) = (\lambda - \tau)\frac{q(\omega, t)}{Q_t} y(t)L_t.$$

The profit flow from exporting π_E is an increasing function of the per-unit profit margin $\lambda - \tau$, the relative quality of the firm's product $q(\omega, t)/Q_t$ and the market size measure $y(t)L_t$.

Since it becomes common knowledge how to produce a good after a higher quality version is discovered, any firm can produce and sell it. It follows that competitive fringe firms price at marginal cost and earn zero profits. Therefore all products are either sold by leaders at price λ or sold by the competitive fringe at price 1.

2.4 R&D Races and the R&D Cost to Becoming an Exporter.

There is two R&D activities described by two distinct R&D technologies: inventing higher quality levels of existing products and learning how to export. Labor is the only input used in both R&D activities. There are quality leaders, firms that hold the patent for the most advanced product within a certain product variety and followers, firm that try to improve upon the products that are sold by leaders. We solve for an equilibrium where Home firms do not improve on products originating from Foreign and Foreign firms do not improve on products originating from Home.

Leaders that produce for the local market do not try to improve on their own products.

Given the same R&D technology as that of followers, they have a smaller incentive to innovate in comparison to followers. A non-exporting leader has strictly less to gain $\pi_L(j+1) - \pi_L(j)$ from improving on its own product (omitting ω and t for brevity) compared to a follower who would gain $\pi_L(j+1)$, hence leaders can not successfully compete for R&D financing with followers. If a leader is an exporter, the gain will be $\pi_L(j+1) + \pi_E(j+1) - \pi_L(j) - \pi_E(j)$. That gain is lower than that of a follower $\pi_L(j+1)$ if $\delta < 2$. Given $\delta \equiv \lambda^{\frac{\alpha}{1-\alpha}}$, for exporting leaders not to have an incentive to improve on their own products, we must have $\lambda < 2^{\frac{1-\alpha}{\alpha}}$. Limit pricing requires $\lambda < 1/\alpha$ and for firms to be able to export requires $\tau < \lambda$. Hence we can write our final assumption on λ as $\tau < \lambda < \min\left(1/\alpha, 2^{\frac{1-\alpha}{\alpha}}\right)$. This guarantees that exporting leaders do not try to improve their own products.

Followers are the ones that invest in quality improving R&D and once they discover a state-of-the-art quality product, they take over the local market from the previous leader. Let I_i denote the Poisson arrival rate of improved products attributed to follower i's investment in R&D. The innovative R&D technology for follower firm i is given by

$$I_i = \frac{Q_t^{\phi} A_F l_i}{\delta^{j(\omega,t)}},$$

where l_i is the labor devoted to R&D by the follower, $\phi < 1$ is an R&D spillover parameter, and $A_F > 0$ is an R&D productivity parameter. The R&D spillover parameter ϕ can be positive or negative but the restriction $\phi < 1$ is necessary to ensure that the model has a finite equilibrium rate of economic growth. The term $\delta^{j(\omega,t)}$ in the R&D technology captures the idea that as product quality increases over time and products become more complex, further innovation becomes increasingly difficult. Venturini (2010) finds that the R&D-driven growth models with the best empirical support assume increasing R&D difficulty.

The returns to innovative R&D are independently distributed across firms, across product varieties and over time. Summing over all firms, the Poisson arrival rate of improved products attributed to all investment in R&D within a particular product variety ω is given by

$$I \equiv \sum_i I_i = \frac{Q_t^{\phi} A_F l}{\delta^{j(\omega,t)}}$$

where $l \equiv \sum_i l_i$ is the total labor devoted to innovative R&D. We solve the model for an equilibrium where the product innovation rate I does not vary across product varieties $\omega \in [0, 1]$.

The second R&D activity is that of leaders learning how to become exporters. This activity can be seen as learning to comply with foreign market regulations, establishing a distribution network, and more generally, paying for the information needed to adapt to a less familiar environment. Muendler and Molina (2009) provide evidence that firms invest when preparing to enter foreign markets. Hiring workers with previous experience in exporting

firms increases their probability of becoming exporters.

The investment each firm needs to make in R&D labor to enter the foreign market is a type of fixed cost of market entry, a common feature in the heterogenous firm literature. The fixed cost here is stochastic and firms with more sophisticated products need to invest more in order to achieve the same arrival rate of the knowledge on how to enter the foreign market. Leaders invest l_E units of labor in an R&D technology which makes them exporters with an instantaneous probability (or Poisson arrival rate)

$$I_E = \left(\frac{Q_t^{\phi} A_E l_E}{\delta^{j(\omega,t)}} \right)^{\gamma}, \tag{3}$$

where A_E is an R&D productivity parameter, $\gamma \in (0,1)$ measures the degree of decreasing returns to leader R&D expenditure, and ϕ is the same R&D spillover parameter. The term $\delta^{j(\omega,t)}$ appears again in the learning-to-export technology and captures the idea that it is more difficult to learn how to export a more advanced product.

There are four types of firms that sell products within the Home country. First, there are Home leaders who export their products. The measure of product varieties produced by these firms is m_{LE}. Second, there are Home leaders who do not export their products. The measure of product varieties produced by these firms is m_{LN}. Third, there are Foreign exporters. The measure of product varieties produced by these firms is m_{FE}. Fourth, there are competitive fringe firms. If a better version of a product is developed abroad and the new Foreign leader has not yet learned how to export this product, then the next lower quality version of that product is produced at Home by competitive fringe firms. The measure of product varieties produced by these firms is m_{CF}. Since all product varieties from both countries are available to the consumers in each country and there is a measure one of product varieties that consumers buy, it follows that $m_{LN} + m_{LE} + m_{FE} + m_{CF} = 1$ holds. Due to symmetry, the measure of product varieties produced by Home exporters equals the measure of product varieties produced by Foreign exporters, that is, $m_{LE} = m_{FE}$. Furthermore, half of all product varieties are produced by Home leaders at Home and half of all product varieties are produced by Foreign leaders at Foreign, so $m_{LN} + m_{LE} = \frac{1}{2}$ also holds.

Figure 1 below describes what happens with a product sold initially by a non-exporting firm. The state-of-the-art quality is produced by the non-exporting firm and the competitive fringe produces the next lower quality version of the same product abroad. Leaders do not improve on their own products, only followers do. A non-exported product is improved on by some follower at the innovation rate I (lower left arrow). Also, the current non-exporting leader learns how to become an exporter at a rate I_E (lower middle arrow). When the product begins to be exported, the exporting leader takes over the foreign market. Products sold by exporters are state-of-the-art quality in both countries. The competitive fringe knows how to produce a one step lower quality version, but the exporting leader prices in such a way

that it drives the competitive fringe out of business. The exporting leader sells its product both at home and abroad until its product is improved on by a follower at home, which happens at the rate I (upper middle arrow). The new leader takes over the home market and sells the better version there, whereas the older version is sold abroad at marginal cost. The new incumbent at home needs to learn how to export in order to take over the foreign market.

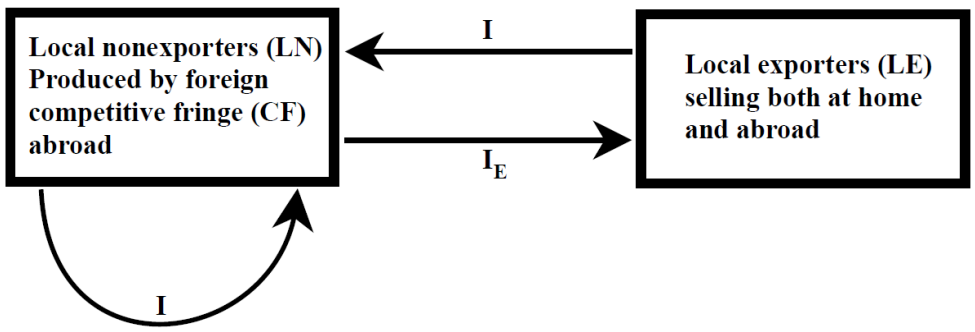

Figure 1: Product Dynamics.

2.5 Bellman Equations and Value Functions

Firms maximize their expected discounted profits. Followers solve a stochastic optimal control problem with a state variable $j(\omega, t)$, which is a Poisson jump process of magnitude one. Non-exporting leaders maximize over the intensity of R&D dedicated to learning how to export, where the knowledge arrives at a certain Poisson rate after which the firm becomes an exporter. The only decision exporters make is over what prices to charge in both markets. Other than that, they exploit the market power they have until a better version of their product ω is discovered by a follower.

Free entry into innovative R&D races and constant returns to scale in the R&D technology together imply that followers have zero market value. Let $v_F(j) = 0$ be the value of a follower when the current state-of-the-art quality is j. All followers have the same zero value regardless of whether they are targeting exporters and non-exporters. Let $v_{LN}(j)$ be the value of a leader that does not export (omitting ω and t from the value function for notational simplicity) and let $v_{LE}(j)$ be the value of a leader that does export.

The Bellman equation for follower firm i is $rv_F(j) = \max_{l_i} -l_i + I_i v_{LN}(j+1)$. The follower invests l_i in R&D and becomes a non-exporting leader with an instantaneous probability I_i. Substituting for I_i from the R&D technology equation and solving gives the following

expression for the value of a non-exporting leader:

$$v_{LN}(j) = \frac{\delta^{j(\omega,t)}}{Q_t^{\phi}\delta A_F}.$$

The value of the firm increases in the quality of the product for which it holds a patent.

The Bellman equation for a non-exporting leader is given by

$$rv_{LN}(j) = \max_{l_E} \pi_L(j) - l_E - Iv_{LN}(j) + I_E\left(v_{LE}(j) - v_{LN}(j)\right) + \dot{v}_{LN}(j). \tag{4}$$

This equation states that the maximized expected return on the non-exporting leader's stock must equal the return on an equal-sized investment in a riskless bond $rv_{LN}(j)$. The return is equal to a stream of profits $\pi_L(j)$ minus investment in R&D to enter the foreign market l_E, plus the arrival rates and respective changes in value attributed to being overtaken by a follower $-Iv_{LN}(j)$ and becoming an exporter $I_E\left(v_{LE}(j) - v_{LN}(j)\right)$, plus the capital gain term $\dot{v}_{LN}(j)$ because the value of the firm can change over time. Non-exporting leaders make a decision over l_E, how much to invest in R&D to learn how to export.

The Bellman equation for an exporting leader is simpler in the sense that exporting firms do not invest in R&D. They only exploit their quality advantage over other firms and the knowledge how to export. They face the risk of being replaced by a firm that learns how to produce a higher quality version of the same product and thus, the Bellman equation for an exporting leader is $rv_{LE}(j) = \pi_L(j) + \pi_E(j) - Iv_{LE}(j) + \dot{v}_{LE}$. The value of an exporting leader is derived from (4), after substituting for $v_{LN}(j)$ and for l_E from (3). We obtain

$$v_{LE}(j) = \frac{\delta^{j(\omega,t)}}{Q_t^{\phi}}\left(\frac{I_E^{\epsilon}}{\gamma A_E} + \frac{1}{\delta A_F}\right), \tag{5}$$

where $\epsilon \equiv (1-\gamma)/\gamma > 0$. The value of an exporter increases in the quality of the product it produces and is also positively related to the rate at which firms become exporters I_E.

2.6 Finding the Labor and R&D Equations

To solve the model, it turns out that a key variable is relative R&D difficulty $x(t) \equiv Q_t^{1-\phi}/L_t$. L_t is the size of the market and $Q_t^{1-\phi}$ is an increasing function of the average quality of all available products. As this average quality increases over time, innovation becomes relatively more difficult. On the other hand, as the size of the market increases, there are more resources that can be devoted to innovation. We will show that solving the model reduces to solving a simple system of two linear equations in two unknowns, where the two unknowns are relative R&D difficulty $x(t)$ and the consumer demand measure $y(t) \equiv Q_t\lambda^{-\sigma}c_t/P_t^{1-\sigma}$. The two equations are the labor equation that describes when there is full employment of labor and the R&D equation that is derived from the profit-maximizing decisions of firms.

To find the labor equation, we need to first introduce some terms connected with product

quality. Given that $Q_t \equiv \int_0^1 q(\omega,t)d\omega$ is the average quality of all products sold in Home, let $Q_{LE} \equiv \int_{m_{LE}} q(\omega,t)d\omega$ be a quality index of products produced by Home leaders that export, $Q_{LN} \equiv \int_{m_{LN}} q(\omega,t)d\omega$ be a quality index of products produced by Home leaders that do not export, $Q_{FE} \equiv \int_{m_{FE}} q(\omega,t)d\omega$ be a quality index of products produced by Foreign exporters, and $Q_{CF} \equiv \int_{m_{CF}} q(\omega,t)d\omega$ be a quality index of products produced by the Home competitive fringe. These quality indexes are all functions of time but this is omitted to simplify notation. They obviously satisfy

$$Q_t = Q_{LE} + Q_{LN} + Q_{FE} + Q_{CF}. \tag{6}$$

Also let $q_{LE} \equiv Q_{LE}/Q_t$, $q_{LN} \equiv Q_{LN}/Q_t$, $q_{FE} \equiv Q_{FE}/Q_t$ and $q_{CF} \equiv Q_{CF}/Q_t$. Each of these terms represents the quality share of a particular group of firms in the total quality index Q_t, where the share is determined not only by the average quality within the group but also by the measure of firms constituting the group. The quality shares satisfy $1 = q_{LE} + q_{LN} + q_{FE} + q_{CF}$ and must be constant over time in any steady-state equilibrium. Given the symmetry condition $Q_{LE} = Q_{FE}$, it follows that

$$1 = 2q_{LE} + q_{LN} + q_{CF}. \tag{7}$$

All labor in the Home country is fully employed in equilibrium and is divided between employment in the production sector $L_P(t)$ and employment in the R&D sector $L_R(t)$.

Starting with $L_P(t)$, demand by Home consumers for a product sold by a Home leader is $d(\omega,t)L_t = \frac{q(\omega,t)\lambda^{-\sigma}c_t}{P_t^{1-\sigma}}L_t = \frac{q(\omega,t)}{Q_t}y(t)L_t$. Demand for an exported product sold abroad is also $d(\omega,t)L_t$, but $\tau d(\omega,t)L_t$ needs to be shipped, and hence $\tau\frac{q(\omega,t)}{Q(t)}y(t)L_t$ is produced. Demand for a product produced by the competitive fringe is $d(\omega,t)L_t = \frac{q(\omega,t)1^{-\sigma}c_t}{P_t^{1-\sigma}}L_t = \frac{q(\omega,t)}{Q_t}y(t)\lambda^\sigma L_t$, where we multiply by λ^σ to take into consideration that the competitive fringe prices at marginal cost, which is one. Thus, total production employment $L_P(t)$ can be expressed as

$$L_P(t) = \int_{m_{LE}+m_{LN}} d(\omega,t)L_t d\omega + \tau\int_{m_{LE}} d(\omega,t)L_t d\omega + \int_{m_{CF}} d(\omega,t)L_t d\omega.$$

Substituting and simplifying gives

$$L_P(t) = \left(q_{LE} + q_{LN} + \tau q_{LE} + \lambda^\sigma q_{CF}\right)y(t)L_t.$$

To solve for employment in the R&D sector, we use the R&D technologies for quality innovation and learning how to export. Rearranging terms yields $l = \frac{I\delta^{j(\omega,t)}}{Q_t^\phi A_F} = q(\omega,t)Q_t^{-\phi}I/A_F$ and $l_E = \frac{I_E^{1/\gamma}\delta^{j(\omega,t)}}{Q_t^\phi A_E} = q(\omega,t)Q_t^{-\phi}I_E^{1/\gamma}/A_E$. For half of all product varieties ($m_{LE}+m_{LN} = 1/2$), Home follower firms do innovative R&D and for varieties with a Home non-exporting leader, these leaders also do R&D to learn how to export. Thus, total R&D employment $L_R(t)$ can

be expressed as

$$L_R(t) = \int_{m_{LE}+m_{LN}} l\, d\omega + \int_{m_{LN}} l_E\, d\omega$$

and after substituting for l and l_E, we obtain

$$L_R(t) = \Big((q_{LE} + q_{LN}) I/A_F \; + \; q_{LN} I_E^{1/\gamma}/A_E \Big)\, x(t) L_t.$$

Full employment of labor implies that $L_t = L_P(t) + L_R(t)$. Dividing both sides by L_t, we obtain the labor equation:

$$1 = (q_{LE} + q_{LN} + \tau q_{LE} + \lambda^\sigma q_{CF})\, y + \Big((q_{LE} + q_{LN}) I/A_F \; + \; q_{LN} I_E^{1/\gamma}/A_E \Big)\, x. \tag{8}$$

In order for equation (8) to hold in steady state equilibrium, it must be the case that $x(t)$ and $y(t)$ are both constant over time, and therefore we will write them as x and y. Once we have solved for the equilibrium values of I, I_E, q_{LE}, q_{LN} and q_{CF}, the labor equation can be graphed as a downward sloping line in (x, y) space (as illustrated in Figure 2). The interpretation of the slope is that when R&D is relatively more difficult (higher x), more resources must be devoted to R&D activities to maintain the steady-state innovation rate and less resources can be devoted to producing goods, so consumer demand y must be lower.

To find the R&D equation, we substitute into (4) for l_E using (3), for $v_{LN}(j)$ and for $v_{LE}(j) - v_{LN}(j)$ using (5). This results in the R&D equation

$$r + I + \phi \frac{\dot{Q}_t}{Q_t} = (\lambda - 1)\delta A_F \frac{y}{x} + \frac{\delta A_F}{A_E} I_E^{1/\gamma} \epsilon. \tag{9}$$

Once we have solved for the steady-state equilibrium values of I, $\dot{Q}_t/Q_t$ and I_E, the R&D equation can be graphed as an upward sloping line in (x, y) space (as illustrated in Figure 2). The interpretation of the slope is that when R&D is relatively more difficult (higher x), consumer demand y must be higher to justify the higher R&D expenditures by firms.

2.7 Quality Dynamics

To determine the steady-state equilibrium innovation rate I, we must first study the dynamics of the different quality indexes.

Since x is constant over time in any steady-state equilibrium, it follows from the definition $x \equiv Q_t^{1-\phi}/L_t$ that $\dot{x}/x = (1-\phi)\dot{Q}_t/Q_t - n = 0$ and $\dot{Q}_t/Q_t = n/(1-\phi)$. Also since q_{LE}, q_{LN} and q_{CF} are all constant over time in any steady-state equilibrium, it follows that $\dot{q}_{LE}/q_{LE} = \dot{Q}_{LE}/Q_{LE} - \dot{Q}_t/Q_t = 0$, and corresponding calculations yield

$$\frac{\dot{Q}_t}{Q_t} = \frac{\dot{Q}_{LE}}{Q_{LE}} = \frac{\dot{Q}_{LN}}{Q_{LN}} = \frac{\dot{Q}_{CF}}{Q_{CF}} = \frac{n}{1-\phi}. \tag{10}$$

In any steady-state equilibrium, the quality indexes of all types of firms must grow at the same rate.

The dynamics of $Q_{LE} \equiv \int_{m_{LE}} \delta^{j(\omega,t)} d\omega$ is given by the differential equation

$$\dot{Q}_{LE} = \int_{m_{LN}} \delta^{j(\omega,t)} I_E d\omega - \int_{m_{LE}} \delta^{j(\omega,t)} I d\omega,$$

where the first integral captures that non-exported products become exported products at the rate I_E, and the second integral captures that exported products become non-exported products when innovation occurs, which happens at the rate I. Using the definitions of the quality indexes and dividing by Q_{LE}, we obtain the growth rate of Q_{LE}:

$$\dot{Q}_{LE}/Q_{LE} = (q_{LN}/q_{LE}) I_E - I.$$

Proceeding in a similar fashion, the dynamics of Q_{LN} is given by the differential equation

$$\dot{Q}_{LN} = \int_{m_{LN}} \left(\delta^{j(\omega,t)+1} - \delta^{j(\omega,t)} \right) I d\omega - \int_{m_{LN}} \delta^{j(\omega,t)} I_E d\omega + \int_{m_{LE}} \delta^{j(\omega,t)+1} I d\omega,$$

where the first integral captures that non-exported products are improved on at the rate I, the second integral captures that non-exporters become exporters at the rate I_E and the third integral captures that exported products are improved upon at the rate I, after which these products become non-exported. This time dividing by Q_{LN}, we obtain

$$\dot{Q}_{LN}/Q_{LN} = (\delta - 1) I - I_E + \delta (q_{LE}/q_{LN}) I.$$

The quality dynamics for the competitive fringe at Home is dependent entirely on the dynamics of firms in Foreign. The inflow of product varieties into the Home competitive fringe is from all Foreign exporters whose products are improved upon at the rate I by Foreign followers. The outflow is from the group of Foreign non-exporters who learn to become exporters at the rate I_E and take back the market of a product previously produced by the Home competitive fringe. Thus, the dynamics of Q_{CF} is given by the differential equation

$$\dot{Q}_{CF} = \int_{m_{LE}} \delta^{j(\omega,t)} I d\omega - \int_{m_{CF}} \delta^{j(\omega,t)} I_E d\omega.$$

Using the definitions of the quality indexes and dividing by Q_{CF}, we obtain

$$\dot{Q}_{CF}/Q_{CF} = (q_{LE}/q_{CF}) I - I_E.$$

Given (10), we can solve the two equations $\dot{Q}_{LE}/Q_{LE} = (q_{LN}/q_{LE}) I_E - I = n/(1-\phi)$ and $\dot{Q}_{LN}/Q_{LN} = (\delta - 1) I - I_E + \delta (q_{LE}/q_{LN}) I = n/(1-\phi)$ for I and then combine them to eliminate the I term. This yields a quadratic equation in q_{LN}/q_{LE} that has only one positive

solution. Plugging this positive solution back into $(q_{LN}/q_{LE})I_E - I = n/(1-\phi)$, we obtain the unique steady-state equilibrium innovation rate:

$$I = \frac{n}{(\delta - 1)(1-\phi)}.$$

The innovation rate I depends in the long run on the population growth rate $n > 0$, the R&D difficulty growth parameter $\delta > 1$ and the intertemporal R&D spillover parameter $\phi < 1$. Individual researchers become less productive with time ($\delta > 1$) and what keeps the innovation rate steady in the long run is the growing number of people employed in the R&D sector, which is made possible by positive population growth ($n > 0$).

Having solved for the steady-state innovation rate I, straightforward calculations lead to the steady-state variety shares q_{LE}, q_{LN} and q_{CF}. We obtain that

$$\begin{aligned} q_{LE} &= \left(2 + \frac{I\delta}{I_E} + \frac{I}{I(\delta-1)+I_E}\right)^{-1} \\ q_{LN} &= q_{LE}\frac{I\delta}{I_E} \\ q_{CF} &= q_{LE}\frac{I}{I(\delta-1)+I_E} \end{aligned}$$

All three variety shares are uniquely determined once we have solved for the steady-state rate at which firms learn how to export I_E.

2.8 Finding I_E

Using the symmetry condition $Q_{FE} = Q_{LE}$, the quality-adjusted price index P_t satisfies $P_t^{1-\sigma} = \int_0^1 q(\omega,t)p(\omega,t)^{1-\sigma}d\omega = Q_{LE}\lambda^{1-\sigma} + Q_{LN}\lambda^{1-\sigma} + Q_{FE}\lambda^{1-\sigma} + Q_{CF} = (2q_{LE}\lambda^{1-\sigma} + q_{LN}\lambda^{1-\sigma} + q_{CF})Q_t$. It follows that $P_t^{1-\sigma}$ must grow at the same rate $n/(1-\phi)$ as Q_t in any steady-state equilibrium. We have already established that $y \equiv Q_t\lambda^{-\sigma}c_t/P_t^{1-\sigma}$ is constant over time, so it immediately follows that consumer expenditure c_t must be constant over time. Thus, the consumer optimization condition $\dot{c}_t/c_t = r - \rho$ implies that $r = \rho$ holds.

Using the Bellman equation for an exporting leader, substituting for $\pi_L(j)$ and $\pi_E(j)$, for $v_{LE}(j)$ using (5) and then substituting into the R&D equation (9) for y/x, we obtain

$$\frac{1}{\delta A_F} = \frac{\lambda - 1}{2\lambda - 1 - \tau}\left(I_E^{\epsilon}\frac{1}{\gamma A_E} + \frac{1}{\delta A_F}\right) + \frac{I_E^{\frac{1}{\gamma}}\epsilon}{A_E\left(r + I + \phi\dot{Q}_t/Q_t\right)}. \tag{11}$$

Taking into account that $r = \rho$, $I = \frac{n}{(\delta-1)(1-\phi)}$, and $\dot{Q}_t/Q_t = \frac{n}{1-\phi}$, the RHS of (11) is a monotonically increasing function of I_E. Thus equation (11) uniquely determines the steady-state equilibrium value of I_E. Furthermore, since the RHS decreases when τ falls holding I_E fixed, I_E must increase to restore equality in (11). We have established one of

the central results in this paper:

Proposition 1 *Trade liberalization induces a higher level of investment in learning how to export ($\tau \downarrow \implies I_E \uparrow$).*

This result is quite intuitive. When the barriers to trade are decreased, it becomes more profitable to be an exporter. Therefore firms invest more in learning how to export.

2.9 The Steady State Equilibrium

Given that we have solved for the steady-state equilibrium values of I, I_E, q_{LN}, q_{LE} and q_{CF}, the labor equation (8) can be graphed as a downward sloping line in (x, y) space. Given that we have also solved for the steady-state equilibrium values of r and $\dot{Q}_t/Q_t$, the R&D equation (9) can be graphed as an upward sloping line in (x, y) space. Both equations are illustrated in Figure 2 and keeping in mind that x and y are constant in steady state, the unique intersection of these two equilibrium conditions at point A determines the steady-state values of relative R&D difficulty x and consumer demand y.

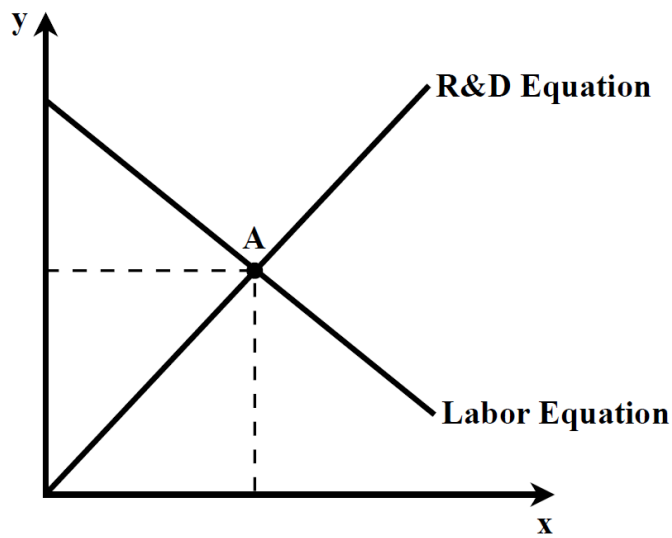

Figure 2: The Steady-State Equilibrium.

We can determine the rate of economic growth in this steady-state equilibrium by studying how consumer utility changes along the equilibrium path. Substituting (2) into (1) and using $y \equiv Q_t \lambda^{-\sigma} c_t / P_t^{1-\sigma}$ to substitute for c_t, we obtain

$$u_t = y\lambda^{\sigma} Q_t^{\frac{1}{\sigma-1}} \left[(2q_{LE} + q_{LN})\lambda^{1-\sigma} + q_{CF} \right]^{\frac{\sigma}{\sigma-1}} . \tag{12}$$

Taking logs and differentiating the above expression with respect to time gives the utility

growth rate $g_u \equiv \dot{u}_t/u_t = \frac{1}{\sigma-1}\dot{Q}_t/Q_t$, which after substituting for $\dot{Q}_t/Q_t$ yields

$$g_u \equiv \dot{u}_t/u_t = \frac{n}{(\sigma-1)(1-\phi)}. \tag{13}$$

The utility growth rate is proportionate to the population growth rate n. Since static utility u_t is proportional to consumer expenditure c_t and static utility increases over time only because $Q_t^{1/(\sigma-1)}$ increases, $Q_t^{1/(\sigma-1)}$ is a measure of the real wage at time t. Thus the real wage growth rate is the same as the utility growth rate and g_u also represents the rate of economic growth in this model.

Equation (13) implies that public policy changes like trade liberalization (a decrease in τ) have no effect on the steady-state rate of economic growth. In this model, growth is "semi-endogenous." We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1870 to 1995, Jones (2005, Table 1) shows that a simple linear trend fits the data extremely well. Further evidence for equation (13) is provided by Venturini (2010). Looking at US manufacturing industry data for the period 1973-1996, he finds that semi-endogenous growth models (where public policies do not have long-run growth effects) have better empirical support than fully-endogenous growth models (where public policies have long-run growth effects).

For the measures m_{LN} and m_{LE} to remain constant in steady-state equilibrium, the outflow of firms from m_{LN} must be equal to the inflow, that is, $m_{LN}I_E = m_{LE}I$. Substituting for m_{LN} using $m_{LN} + m_{LE} = \frac{1}{2}$ yields $\left(\frac{1}{2} - m_{LE}\right) I_E = m_{LE}I$, from which it follows that $m_{LE} = \frac{I_E/2}{I+I_E}$ and $m_{LN} = \frac{I/2}{I+I_E}$. The last two equations show that an increase in I_E leads to an increase in the measure of products purchased from exporting leaders m_{LE} and a decrease in the measure of products purchased from non-exporting leaders m_{LN}.

2.10 Firm Exit

When a firm innovates and becomes a new quality leader, one can say that the "birth" of a new firm has occurred. This birth is also associated with "death", as the previous quality leader stops producing and in a sense dies. We define the firm exit rate or death rate n_D as the rate at which firms die in the Home country.

To calculate the firm exit rate, we need to first specify how many firms produce a product when it is produced by the competitive fringe. When it becomes common knowledge how to produce a product variety, any firm can produce it. We solve for an equilibrium where two firms actually do. Given that firms are price-setters, two firms is enough to generate a perfectly competitive outcome with zero economic profits (the Bertrand equilibrium), so

there is no incentive for other firms that know how to produce a product to enter and start producing.[1]

The firm exit rate is then given by

$$n_D \equiv \frac{Im_{LN} + Im_{LE} + (I_E + I)2m_{CF}}{m_{LN} + m_{LE} + 2m_{CF}}.$$

For the measure of product varieties $m_{LN}+m_{LE}$ where there are Home quality leaders, Home innovation occurs at the rate I and results in the death of these firms. For the measure of product varieties m_{CF} where there is a Foreign non-exporting leader and a Home competitive fringe (consisting of two producers), both Foreign innovation (which occurs at rate I) and Foreign learning how to export (which occurs at rate I_E) result in the death of the current Home producers.

Using $m_{LE} = \frac{I_E/2}{I+I_E}$ and $m_{LN} = \frac{I/2}{I+I_E} = m_{CF}$, straightforward calculations yield the steady-state firm exit rate

$$n_D = \frac{3I\left(I + I_E\right)}{3I + I_E}.$$

Since $\partial n_D/\partial I_E = 6I^2/\left(3I + I_E\right)^2 > 0$, it follows that trade liberalization leads to a higher rate at which firms die, since trade liberalization increases I_E. We have established

Proposition 2 *Trade liberalization leads to a higher firm exit rate ($\tau \downarrow \Longrightarrow n_D \uparrow$).*

Pavcnik (2002) studies a period of trade liberalization in Chile (1979-1986) and reports that it coincided with a "massive" exit rate of firms. Gibson and Harris (1996) present evidence of increasing firm exit as a result of trade liberalization in New Zealand. Gu, Sawchuk and Rennison (2003) show a significant increase in the exit rate of firms in 81 Canadian manufacturing industries as a result of tariff cuts. Initially lower exit rates increased after trade liberalization policies were introduced. This paper presents the first model that is consistent with this evidence. Haruyama and Zhao (2008) present another quality ladders growth model with endogenous firm turnover but trade liberalization does not affect the firm exit rate in their setup. In Melitz (2003), an exogenous firm exit rate is assumed (since there is no other reason why firms would choose to go out of business) and consequently trade liberalization has no effect on the exit rate of firms that have already entered a market.

[1] This is a model with an infinite number of equilibria. In markets where firms in the competitive fringe produce, the number of producing firms can be two, three or one billion. Given that the equilibrium price equals marginal cost and producing firms earn zero profits, competitive fringe firms are indifferent between producing and not producing. However, all the equilibria look the same from the perspective of consumers and thus we choose to focus on one particular equilibrium, the one where two firms from the competitive fringe produce when a product is produced by the competitive fringe. If only one competitive fringe firm produces a product, then this firm maximizes its profits by charging a price greater than marginal cost. Other firms then have an incentive to enter and charge a slightly lower price, so there is no equilibrium with just one firm from the competitive fringe producing.

2.11 Comparing Exporters and Non-Exporters

We now examine whether exporting firms charge higher prices than non-exporting firms. The average price charged by exporting firms is $P_E \equiv \frac{m_{LE}\lambda + m_{FE}\lambda}{m_{LE}+m_{FE}} = \lambda$. The average price charged by non-exporting firms is $P_N \equiv \frac{m_{LN}\lambda + 2m_{CF}1}{m_{LN}+2m_{CF}}$ and thus $P_E > P_N$ always holds. We have established

Proposition 3 *Exporting firms charge higher prices on average than non-exporting firms.*

A number of recent papers point out the correlation of export status with prices charged by firms. Kugler and Verhoogen (2008) use data from Colombia to compare output prices (what firms charge on their home markets) and export status of manufacturers. They find a positive relationship, that is, exporters charge higher prices. Hallak and Sivadasan (2009) also find a positive relationship using Indian and U.S. data. In our model, exporters charge the price λ and this is higher than the average price of non-exporters, which is a convex combination of the price λ charged by non-exporting leaders and the price one charged by competitive fringe firms.

The Melitz (2003) model cannot account for the above-mentioned evidence regarding the pricing behavior of exporters and non-exporters. In Melitz (2003), it is the firms that charge the lowest prices that export. The firms that charge the lowest prices are the highest productivity firms and the highest productivity firms are the firms that export.

Baldwin and Harrigan (2007) develop an alternative model to account for the evidence about the pricing behavior of exporters. In their model, any firm that draws a higher marginal cost can also produce a higher quality product. The competitiveness of firms increases with higher marginal cost due to the lower quality-adjusted price that they charge. Baldwin and Harrigan assume that $q = a^{1+\theta}$, where q is the quality level of a product, a its marginal cost and θ is a parameter that is restricted to be positive. Given $\theta > 0$, quality increases quickly enough so that the quality-adjusted price falls as marginal cost increases. Exporters end up producing higher quality products and charging higher prices. In our model by contrast, all firms have the same marginal cost of one and there is no connection between marginal cost and the quality of products. Nevertheless, our model is consistent with the evidence that exporters tend to charge higher prices.

De Loecker and Warzynski (2012) study a panel of Slovenian firms for the period 1994-2000 and find that exporters charge significantly higher markups (of price over marginal cost). Their measure of markups for exporters is a share weighted average markup across markets, where the weight by market is the share of an input's expenditure used in production sold in that market. Using that definition in the setup of our model, the markup of the average exporting firm is $\mu_E \equiv \frac{\tau}{1+\tau}\frac{\lambda}{\tau} + \frac{1}{1+\tau}\frac{\lambda}{1}$, which simplifies to $\mu_E = \frac{2\lambda}{1+\tau}$. Non-exporting firms are either non-exporting leaders or competitive fringe firms (with two producing in equilibrium).

The markup of the average non-exporting firm is $\mu_N \equiv \frac{m_{LN}}{m_{LN}+2m_{CF}}\frac{\lambda}{1} + \frac{2m_{CF}}{m_{LN}+2m_{CF}}\frac{1}{1}$. Using $m_{LN} = m_{CF}$, this simplifies to $\mu_N = \frac{\lambda+2}{3}$. Thus our model is consistent with the evidence in De Loecker and Warzynski (2012) and satisfies $\mu_E > \mu_N$ if $\frac{2\lambda}{1+\tau} > \frac{\lambda+2}{3}$. It is easy to show that this is the case for all $\lambda \in (1,2)$ and with the focus in this paper on non-drastic innovations, $\lambda \in (1,2)$ is easily satisfied.[2] We can therefore write:

Proposition 4 *Exporting firms charge higher markups on average than non-exporting firms.*

We can also examine whether exporting firms are more productive than non-exporting firms. For all firms, one unit of labor produces one unit of output but firms differ in the quality of products they know how to produce. Thus, more productive firms in our model are firms that know how to produce higher quality products. The average quality of products produced by exporting firms is $Q_E \equiv \frac{Q_{LE}+Q_{FE}}{m_{LE}+m_{FE}} = \frac{Q_{LE}}{m_{LE}} = \frac{q_{LE}}{m_{LE}}Q_t$. The average quality of products produced by non-exporting firms is $Q_N \equiv \frac{Q_{LN}+2Q_{CF}}{m_{LN}+2m_{CF}} = \frac{q_{LN}+2q_{CF}}{m_{LN}+2m_{CF}}Q_t = \frac{q_{LN}+2q_{CF}}{3m_{LN}}Q_t$ since $m_{LN} = m_{CF}$. It is straightforward to verify that exporting firms sell higher quality products on average than non-exporting firms and hence have higher average productivity when I_E is sufficiently low:

Proposition 5 *Exporting firms are more productive on average than non-exporting firms and $Q_E > Q_N$ holds if $3 > \delta$ and $(3-\delta)I > I_E$.*

The condition $3 > \delta$ is easily satisfied for plausible parameter values. For example, if $\lambda = 1.40$, $\alpha = 0.6$ and $\sigma = 1/(1-\alpha) = 2.5$, then $3 > \delta \equiv \lambda^{\sigma-1} = 1.4^{1.5} \approx 1.65$. Equation (11) implies that $(3-\delta)I > I_E$ is satisfied when A_E is sufficiently small because then I_E is sufficiently small. The condition $(3-\delta)I > I_E$ holds when the product innovation rate I is significantly higher that the rate at which firms learn to become exporters I_E and most firms are non-exporters in equilibrium. This is exactly what Bernard et. al. (2003) find in their study of 200,000 U.S. manufacturing plants, where only 21 percent reported exporting. Thus, we view $(3-\delta)I > I_E$ as being the main case of interest, and when this condition holds, the model has the implication that exporting firms are more productive on average than non-exporting firms.

In Melitz (2003), not only are exporting firms more productive on average than non-exporting firms, all exporting firms are more productive than all non-exporting firms. There is a threshold productivity value which separates exporters from non-exporters, with all exporters having productivity above the threshold and all non-exporters having productivity below the threshold. In our model by contrast, there is no such threshold: there are exporters that are less productive than certain non-exporters. Proposition 5 speaks about

[2] The inequality $\frac{2\lambda}{1+\tau} > \frac{\lambda+2}{3}$ can be rewritten as $5\lambda > \lambda\tau + 2\tau + 2$. Since $\tau < \lambda$, the inequality holds if $5\lambda > \lambda^2 + 2\lambda + 2$ or $0 > (\lambda-2)(\lambda-1)$. We conclude that $\mu_E > \mu_N$ holds for all $\lambda \in (1,2)$.

productivity on average within the groups of exporters and non-exporters. In support of this proposition, Bernard et. al. (2003) present empirical evidence that the exporter productivity distribution is substantially shifted to the right (higher productivity) compared to the non-exporter productivity distribution, but at the same time there is a significant overlap in these distributions, meaning that there does not exist a threshold productivity value separating exporters from non-exporters.

3 Numerical Results

To learn more about the steady-state equilibrium properties of the model, we turn to computer simulations. In this section, we report results obtained from solving the model numerically.

In our computer simulations, we used the following benchmark parameter values: $\rho = 0.04$, $n = 0.014$, $\tau = 1.3$, $\lambda = 1.4$, $\alpha = 0.6$, $\gamma = 0.5$, $\phi = 0.53$, $L_0 = 1$, $A_F = 1$ and $A_E = 0.59$. The subjective discount rate ρ was set at 0.04 to reflect a real interest rate of 4 percent, consistent with evidence in McGrattan and Prescott (2005). The population growth rate $n = 0.014$ equals the annual rate of world population growth between 1991 and 2000 according to the World Development Indicators (World Bank, 2003). Novy (2011) estimates that the year 2000 tariff equivalent of US trade costs with Canada and Mexico were 25 percent and 33 percent, respectively. The trade cost parameter τ was set at 1.3 to reflect 30 percent trade costs. The innovation size parameter choice $\lambda = 1.4$ then implies that the markup of price over marginal cost is 40 percent in the domestic market ($\lambda/1$) and 8 percent in the export market (λ/τ), which is within the range of markup estimates reported in Morrison (1990). The preference parameter α was set at 0.6 to guarantee that innovations are not drastic and the assumption $\lambda < \min\left(1/\alpha, 2^{\frac{1-\alpha}{\alpha}}\right)$ is satisfied. The parameter $\gamma = 0.5$ describes the degree of decreasing returns to R&D in learning how to export and is within the range of decreasing returns to R&D estimates reported in Kortum (1993). The R&D spillover parameter $\phi = 0.53$ was chosen to generate a steady-state economic growth rate of 2 percent using $g_u = \frac{n}{(\sigma-1)(1-\phi)}$, which is consistent with the average US GDP per capita growth rate from 1950 to 1994 reported in Jones (2005). $L_0 = 1$ represents a normalized value for the initial population level at time $t = 0$ and $A_F = 1$ represents a normalized value for the R&D productivity parameter. Finally, $A_E = 0.59$ was chosen to guarantee that 21 percent of firms export, consistent with the evidence in Bernard et. al. (2003). With these benchmark parameter choices, the condition $(3 - \delta)I > I_E$ is satisfied and exporting firms are more productive on average than non-exporting firms.

To solve the model, we first solve (11) for the steady-state equilibrium value of I_E. Then we solve simultaneously the labor equation (8) and the R&D equation (9) for the steady-state equilibrium values of x and y. The results obtained from solving the model numerically are

reported in Table 1.[3] The top row of results shows the steady state equilibrium outcome for the benchmark parameter values (including $\tau = 1.3$) and the remaining rows show how the steady state equilibrium changes when the trade cost parameter τ is decreased, that is, when trade liberalization occurs. The last two columns show the fraction of firms that export $f_E \equiv \frac{m_{LE}}{m_{LE}+m_{LN}+2m_{CF}}$ and the steady-state utility level u_0 of the representative consumer at time $t = 0$. The later is obtained by solving $x \equiv Q_t^{1-\phi}/L_t$ for Q_t, then substituting this expression into (12) and evaluating at time $t = 0$. There are three main conclusions that we draw from studying Table 1.

τ	g_u	I	I_E	q_{LE}	q_{LN}	q_{CF}	x	y	n_D	f_E	u_0
1.3	0.02	0.045	0.036	0.210	0.435	0.144	4.66	0.685	0.064	0.21	7.07
1.2	0.02	0.045	0.061	0.268	0.329	0.134	5.06	0.693	0.073	0.31	7.96
1.1	0.02	0.045	0.079	0.297	0.282	0.124	5.57	0.702	0.079	0.37	9.12
1.0	0.02	0.045	0.093	0.314	0.255	0.116	6.12	0.710	0.082	0.41	10.48

Table 1. The Effects of Trade Liberalization ($\tau \downarrow$)

First, trade liberalization monotonically increases the steady-state rate at which firms learn how to become exporters (when τ decreases from 1.3 to 1.0, I_E increases from 0.036 to 0.093). This property was already established in Proposition 1 but it is interesting to see how large quantitatively the effect is. Due to the increased investment by firms in learning how to export, there is a big increase in the steady-state fraction of firms that export (f_E increases from 0.21 to 0.41), a big increase in the death rate of firms because other firms are learning how to export (n_D increases from 0.064 to 0.082) and a big increase in the quality share of Home exporters in the total quality index (q_{LE} increases from 0.210 to 0.314). The intuition behind these properties is quite straightforward: trade liberalization leads to higher profits from exporting and increases the incentives firms have to learn how to export. Firms respond by devoting more resources to learning how to export and more firms end up exporting in steady-state equilibrium.

Second, trade liberalization monotonically increases the steady-state level of relative R&D difficulty (when τ decreases from 1.3 to 1.0, x increases from 4.66 to 6.12). Since relative R&D difficulty $x(t) \equiv Q_t^{1-\phi}/L_t$ only gradually adjusts over time and a new higher steady-state level means that along the transition path $Q_t^{1-\phi}$ must grow at a higher rate than $L_t = L_0 e^{nt}$, trade liberalization must lead to a temporary increase in the innovation rate. Trade liberalization has no effect on the steady-state innovation rate $I = n/[(\delta - 1)(1 - \phi)] = 0.045$ but the increase in x means that it does lead to a significant temporary increase in innovation by firms.

[3] The MATLAB code used to solve the model can be obtained from the authors upon request.

Third, trade liberalization monotonically increases steady-state consumer utility and aggregate productivity (when τ decreases from 1.3 to 1.0, u_0 increases from 7.07 to 10.48). Thus, the model is consistent with the evidence reported in Pavcnik (2002), Trefler (2004) and Bernard and Jensen (2004b) that trade liberalization leads to aggregate productivity gains. Since the steady-state rate of economic growth $g_u \equiv \frac{n}{(\sigma-1)(1-\phi)} = 0.02$ is unaffected by trade liberalization, we conclude that trade liberalization makes consumers in both countries substantially better off in the long run. The reason why the welfare gains from trade liberalization are so large (a 48 percent increase in u_0) is that trade liberalization benefits consumers through two channels: consumers benefit from the substantial increase in exporting by foreign firms (they are eventually buying a much higher share of imported products and these products are of higher quality than what the local competitive fringe can produce) and consumers benefit from the substantial increase in innovation by firms (they are eventually buying much higher quality versions of domestically produced varieties).

Gustafsson and Segerstrom (2010) present a growth model that is similar to the one in this paper (growth is semi-endogenous) but with Melitz-type assumptions. It can be characterized as a "Melitz model with economic growth" since it has a strictly positive steady-state rate of economic growth and the original Melitz model has a stationary steady-state equilibrium (aggregate variables do not change over time). Gustafsson and Segerstrom solve their model analytically and derive a result that is both surprising and disturbing: when the steady-state rate of economic growth is sufficiently high, trade liberalization retards economic growth and makes all consumers in both countries worse off in the long run (Theorem 3). Trade liberalization benefits consumers through the increase in exporting but leads to a global decrease in innovation by firms. For plausible parameter values and reasonable rates of economic growth (a 2 percent steady-state rate of economic growth, consistent with the evidence in Jones (2005)), the second channel dominates. In the current model however for parameter values that satisfy the same stylized facts, trade liberalization substantially promotes economic growth and makes consumers better off in the long run.

4 Conclusions

In this paper, we build a standard quality ladders endogenous growth model with one important new assumption, firms learn how to export and learning takes time. The R&D process generates an endogenous distribution of firms with heterogeneous productivities. Without Melitz-type assumptions, the model can account for all the evidence that the Melitz (2003) model was designed to explain plus much evidence that the Melitz model cannot account for. Consistent with the empirical evidence, we find that trade liberalization leads to a higher exit rate of firms, that exporters charge higher prices and markups for the products they sell, and that many relatively large firms do not export.

To keep our analysis as simple as possible, we have made some strong assumptions in this paper. We have assumed that all innovations are non-drastic, so no firm can get away with charging a pure monopoly price without losing consumers to rival firms. We have also focused on the case where exporting firms do not try to improve their own products, by making appropriate restrictions on the possible values of model parameters. Exploring how the model's properties change when these simplifying assumptions are relaxed is an important topic for further research.

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Appendix (For Online Publication)

Consumers

The representative household's optimization problem can be solved in three steps.

The first step is to solve the within-variety static optimization problem. Focusing on a particular variety ω, since products of different quality are perfect substitutes by assumption (equation 1), consumers only buy the product(s) with the lowest quality-adjusted price $p(j,\omega,t)/\lambda^j$. The easiest way to see this is to solve the simple consumer optimization problem $\max_{d_1,d_2} d_1 + \lambda d_2$ subject to $p_1 d_1 + p_2 d_2 = c$, $d_1 \geq 0$ and $d_2 \geq 0$. The solution is to only buy good 1 if $p_1 < p_2/\lambda$ and only buy good 2 if $p_1 > p_2/\lambda$.

The second step is to solve the across-variety static optimization problem

$$\max_{d()} \int_0^1 \left[\lambda^{j(\omega,t)} d(\omega,t)\right]^\alpha d\omega \qquad \text{subject to} \qquad c_t = \int_0^1 p(\omega,t) d(\omega,t) d\omega,$$

where $j(\omega,t)$ is the quality level with the lowest quality adjusted price $p(j,\omega,t)/\lambda^j$ of product variety ω at time t, or alternatively, the number of innovations in variety ω from time 0 to time t. This problem can be rewritten as the optimal control problem

$$\max_{d()} \int_0^1 \left[\lambda^{j(\omega,t)} d(\omega,t)\right]^\alpha d\omega \text{ s.t. } \frac{\partial z(\omega,t)}{\partial \omega} = p(\omega,t) d(\omega,t),\ z(0,t) = 0,\ z(1,t) = c_t,$$

where $z(\omega,t)$ is a new state variable. The Hamiltonian function for this optimal control problem is $H \equiv \left[\lambda^{j(\omega,t)} d(\omega,t)\right]^\alpha + \mu(\omega,t) p(\omega,t) d(\omega,t)$, where $\mu(\omega,t)$ is the costate variable. The costate equation $\partial H/\partial z = 0 = -\partial\mu/\partial\omega$ implies that $\mu(\omega,t)$ is constant across ω. Taking this into account, the first-order condition $\partial H/\partial d = \alpha\lambda^{j(\omega,t)\alpha} d(\omega,t)^{\alpha-1} + \mu(t)p(\omega,t) = 0$ implies that $d(\omega,t) = \left(\frac{-\mu(t)p(\omega,t)}{\alpha\lambda^{j(\omega,t)\alpha}}\right)^{1/(\alpha-1)}$. Substituting this expression back into the budget constraint yields

$$c_t = \int_0^1 p(\omega,t) \left(\frac{-\mu(t)p(\omega,t)}{\alpha\lambda^{j(\omega,t)\alpha}}\right)^{\frac{1}{\alpha-1}} d\omega = \left(\frac{-\mu(t)}{\alpha}\right)^{\frac{1}{\alpha-1}} \int_0^1 \left(\frac{p(\omega,t)}{\lambda^{j(\omega,t)}}\right)^{\frac{\alpha}{\alpha-1}} d\omega,$$

from which it follows that

$$\left(\frac{-\mu(t)}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{c_t}{\int_0^1 \left(\frac{p(\omega,t)}{\lambda^{j(\omega,t)}}\right)^{\frac{\alpha}{\alpha-1}} d\omega}.$$

Substituting this expression back into consumer demand yields:

$$\begin{aligned} d(\omega,t) &= \left(\frac{-\mu(t)p(\omega,t)}{\alpha\lambda^{j(\omega,t)\alpha}}\right)^{1/(\alpha-1)} \\ &= \frac{p(\omega,t)^{\frac{1}{\alpha-1}}c_t}{\lambda^{j(\omega,t)\frac{\alpha}{\alpha-1}}\int_0^1\left(\frac{p(\omega,t)}{\lambda^{j(\omega,t)}}\right)^{\frac{\alpha}{\alpha-1}}d\omega} \\ &= \frac{\lambda^{j(\omega,t)\frac{\alpha}{1-\alpha}}p(\omega,t)^{\frac{1}{\alpha-1}}c_t}{\int_0^1\lambda^{j(\omega,t)\frac{\alpha}{1-\alpha}}p(\omega,t)^{\frac{\alpha}{\alpha-1}}d\omega}. \end{aligned}$$

Now $\sigma \equiv \frac{1}{1-\alpha}$ implies that $1-\sigma = \frac{1-\alpha-1}{1-\alpha} = \frac{\alpha}{\alpha-1}$. Also $q(\omega,t) \equiv \delta^{j(\omega,t)} = \lambda^{j(\omega,t)(\sigma-1)} = \lambda^{j(\omega,t)\frac{\alpha}{1-\alpha}}$. Thus

$$d(\omega,t) = \frac{q(\omega,t)p(\omega,t)^{-\sigma}c_t}{\int_0^1 q(\omega,t)p(\omega,t)^{1-\sigma}d\omega}$$

and given that the price index P_t satisfies $P_t^{1-\sigma} = \int_0^1 q(\omega,t)p(\omega,t)^{1-\sigma}d\omega$, we can write the demand function more simply as

$$d(\omega,t) = \frac{q(\omega,t)p(\omega,t)^{-\sigma}c_t}{P_t^{1-\sigma}}. \tag{2}$$

The third step is to solve for the path of consumer expenditure that maximizes discounted utility subject to the relevant intertemporal budget constraint. Static utility satisfies

$$\begin{aligned} u_t &\equiv \left[\int_0^1\left(\sum_j \lambda^j d(j,\omega,t)\right)^\alpha d\omega\right]^{\frac{1}{\alpha}} \\ &= \left[\int_0^1\left(\lambda^{j(\omega,t)}d(\omega,t)\right)^\alpha d\omega\right]^{\frac{1}{\alpha}} \\ &= \left[\int_0^1\left(\lambda^{j(\omega,t)}\frac{q(\omega,t)p(\omega,t)^{-\sigma}c_t}{P_t^{1-\sigma}}\right)^\alpha d\omega\right]^{\frac{1}{\alpha}} \\ &= \frac{c_t}{P_t^{1-\sigma}}\left[\int_0^1\lambda^{j(\omega,t)\frac{\alpha(1-\alpha)}{1-\alpha}}\lambda^{j(\omega,t)\frac{\alpha\alpha}{1-\alpha}}p(\omega,t)^{-\sigma\alpha}d\omega\right]^{\frac{1}{\alpha}} \\ &= \frac{c_t}{P_t^{1-\sigma}}\left[\int_0^1 q(\omega,t)p(\omega,t)^{1-\sigma}d\omega\right]^{\frac{1}{\alpha}} \\ &= \frac{c_t}{P_t^{1-\sigma}}P_t^{(1-\sigma)/\alpha} = \frac{c_t}{P_t^{1-\sigma}}P_t^{-\sigma} = \frac{c_t}{P_t} \end{aligned}$$

given that $-\sigma\alpha = -\frac{\alpha}{1-\alpha} = 1-\sigma$, so $\ln u_t = \ln c_t - \ln P_t$ and $U \equiv \int_0^\infty e^{-(\rho-n)t}\ln[u_t]\,dt = \int_0^\infty e^{-(\rho-n)t}\ln c_t\,dt - \int_0^\infty e^{-(\rho-n)t}\ln P_t\,dt$. Each household chooses the time path of consumer expenditure c_t taking the time paths of $q(\omega,t)$, $p(\omega,t)$ and P_t as given, so the second integral

with $\ln P_t$ can be ignored when maximizing discounted utility. The household's intertemporal optimization problem simplifies to $\max_c \int_0^\infty e^{-(\rho-n)t} \ln c_t dt$ subject to the intertemporal budget constraint $\dot{a}_t = w + (r_t - n)a_t - c_t$, where a_t is the representative consumer's asset holding and w is the wage rate. The Hamiltonian function for this optimal control problem is $H \equiv e^{-(\rho-n)t} \ln c_t + \mu_t(w_t + (r_t - n)a_t - c_t)$ where μ_t is the relevant costate variable. Maximizing the Hamiltonian with respect to the control yields $\partial H/\partial c_t = e^{-(\rho-n)t} c_t^{-1} - \mu_t = 0$, from where we obtain $\mu_t = e^{-(\rho-n)t} c_t^{-1}$. Taking logs and differentiating then yields $\dot{\mu}_t/\mu_t = n - \rho - \dot{c}_t/c_t$. From the costate equation $\partial H/\partial a_t = \mu_t(r_t - n) = -\dot{\mu}_t$, we obtain $\dot{\mu}_t/\mu_t = n - r_t$. Combining the last two results gives the standard Euler equation $\dot{c}_t/c_t = r_t - \rho$.

Product Markets

Letting p denote the price that the firm charges, the profit flow earned by a leader that sells locally is $\pi_L(\omega, t) = (p-1)d(\omega, t)L_t = (p-1)\frac{q(\omega,t)p^{-\sigma}c_t}{P_t^{1-\sigma}} L_t = (p^{1-\sigma} - p^{-\sigma})\frac{q(\omega,t)c_t L_t}{P_t^{1-\sigma}}$. Maximizing π_L with respect to p yields the first order condition

$$\frac{\partial \pi_L(\omega, t)}{\partial p} = [(1-\sigma)p^{-\sigma} + \sigma p^{-\sigma-1}]\frac{q(\omega,t)c_t L_t}{P_t^{1-\sigma}} = p^{-\sigma}[1 - \sigma + \sigma p^{-1}]\frac{q(\omega,t)c_t L_t}{P_t^{1-\sigma}} = 0,$$

from which it follows that the monopoly price is $p = \frac{\sigma}{\sigma-1} = \frac{1}{1-\alpha}/\frac{\alpha}{1-\alpha} = \frac{1}{\alpha}$. But we have assumed that $\lambda < \frac{1}{\alpha}$, so the leader finds it optimal to charge the limit price $p = \lambda$ instead. If the leader charged the monopoly price, it would lose all consumers to the local competitive fringe. Thus $\pi_L(\omega, t) = (\lambda - 1)\frac{q(\omega,t)\lambda^{-\sigma}c_t}{P_t^{1-\sigma}} L_t = (\lambda - 1)\frac{q(\omega,t)}{Q_t}\frac{Q_t\lambda^{-\sigma}c_t}{P_t^{1-\sigma}} L_t$ and letting $y(t) \equiv \frac{Q_t\lambda^{-\sigma}c_t}{P_t^{1-\sigma}}$, we obtain

$$\pi_L(\omega, t) = (\lambda - 1)\frac{q(\omega, t)}{Q_t} y(t) L_t.$$

The profit flow that a leader earns from exporting is $\pi_E(\omega, t) = (p - \tau)d(\omega, t)L_t = (p - \tau)\frac{q(\omega,t)p^{-\sigma}c_t}{P_t^{1-\sigma}} L_t = (p^{1-\sigma} - \tau p^{-\sigma})\frac{q(\omega,t)c_t L_t}{P_t^{1-\sigma}}$, where p now denotes the price charged to consumers in the export market. Maximizing π_E with respect to p yields the first order condition

$$\frac{\partial \pi_E(\omega, t)}{\partial p} = [(1-\sigma)p^{-\sigma} + \tau\sigma p^{-\sigma-1}]\frac{q(\omega,t)c_t L_t}{P_t^{1-\sigma}} = p^{-\sigma}[1 - \sigma + \tau\sigma p^{-1}]\frac{q(\omega,t)c_t L_t}{P_t^{1-\sigma}} = 0,$$

from which it follows that the monopoly price is $p = \frac{\tau\sigma}{\sigma-1} = \frac{\tau}{\alpha}$. But $\lambda < \frac{1}{\alpha} < \frac{\tau}{\alpha}$ for all $\tau > 1$, so the leader also finds it optimal to charge the limit price $p = \lambda$ in the export market. Thus $\pi_E(\omega, t) = (\lambda - \tau)\frac{q(\omega,t)\lambda^{-\sigma}c_t}{P_t^{1-\sigma}} L_t = (\lambda - \tau)\frac{q(\omega,t)}{Q_t}\frac{Q_t\lambda^{-\sigma}c_t}{P_t^{1-\sigma}} L_t$ and given $y(t) \equiv \frac{Q_t\lambda^{-\sigma}c_t}{P_t^{1-\sigma}}$, we obtain

$$\pi_E(\omega, t) = (\lambda - \tau)\frac{q(\omega, t)}{Q_t} y(t) L_t.$$

R&D Races

The condition for exporting leaders to not improve their own products is

$$\begin{aligned}
\pi_L(j+1) &> \pi_L(j+1) + \pi_E(j+1) - \pi_L(j) - \pi_E(j) \\
0 &> \pi_E(j+1) - \pi_L(j) - \pi_E(j) \\
0 &> (\lambda-\tau)\frac{q(j+1)}{Q_t}y(t)L_t - (\lambda-1)\frac{q(j)}{Q_t}y(t)L_t - (\lambda-\tau)\frac{q(j)}{Q_t}y(t)L_t \\
0 &> \frac{y(t)L_t}{Q_t}\left[(\lambda-\tau)q(j+1) - (\lambda-1)q(j) - (\lambda-\tau)q(j)\right] \\
0 &> (\lambda-\tau)\delta^{j+1} - (\lambda-1)\delta^j - (\lambda-\tau)\delta^j \\
0 &> (\lambda-\tau)\delta - (\lambda-1) - (\lambda-\tau).
\end{aligned}$$

Knowing that $\delta > 1$, it is clear that increasing τ decreases the right-hand-side (RHS) of the last expression and makes it easier to satisfy the inequality. We want this inequality to hold in the most restrictive case where free trade holds, namely $\tau = 1$. After dividing $0 > (\lambda-1)\delta - (\lambda-1) - (\lambda-1)$ by $(\lambda-1)$, we obtain $0 > \delta - 2$ or $\delta \equiv \lambda^{\frac{\alpha}{1-\alpha}} < 2$. Thus $\lambda < 2^{\frac{1-\alpha}{\alpha}}$ guarantees that exporting leaders do not have an incentive to improve their own products for all $\tau \geq 1$.

Bellman Equations and Value Functions

The Bellman equation for follower firm i is

$$rv_F(j) = \max_{l_i} -l_i + I_i v_{LN}(j+1) = \max_{l_i} -l_i + Q_t^{\phi}\frac{A_F l_i}{\delta^{j(\omega,t)}}v_{LN}(j+1).$$

The first order condition for an interior solution is $-1 + Q_t^{\phi}\frac{A_F}{\delta^{j(\omega,t)}}v_{LN}(j+1) = 0$ and solving yields $v_{LN}(j+1) = \frac{\delta^{j(\omega,t)}}{Q_t^{\phi}A_F}$ or

$$v_{LN}(j) = \frac{\delta^{j(\omega,t)}}{Q_t^{\phi}\delta A_F}.$$

The Bellman equation for a non-exporting leader is given by

$$rv_{LN}(j) = \max_{l_E} \pi_L(j) - l_E - Iv_{LN}(j) + I_E\left(v_{LE}(j) - v_{LN}(j)\right) + \dot{v}_{LN}(j). \tag{4}$$

The first-order condition for an interior solution is $-1 + \partial I_E/\partial l_E\left[v_{LE}(j) - v_{LN}(j)\right] = 0$ and given that $I_E = \left(Q_t^{\phi}A_E l_E/\delta^{j(\omega,t)}\right)^{\gamma}$, this becomes

$$-1 + \gamma l_E^{\gamma-1}\left(Q_t^{\phi}A_E/\delta^{j(\omega,t)}\right)^{\gamma}\left[v_{LE}(j) - v_{LN}(j)\right] = 0.$$

We can solve the first-order condition for $v_{LE}(j)$. Noting that $I_E^{1/\gamma}\delta^{j(\omega,t)}/(Q_t^{\phi}A_E) = l_E$ and

letting $\epsilon \equiv (1-\gamma)/\gamma > 0$, we obtain

$$v_{LE}(j) - v_{LN}(j) = (1/\gamma) l_E^{1-\gamma} \left(\delta^{j(\omega,t)}/Q_t^{\phi} A_E\right)^{\gamma}$$

$$v_{LE}(j) = (1/\gamma) I_E^{(1-\gamma)/\gamma} \left(\delta^{j(\omega,t)}/Q_t^{\phi} A_E\right)^{1-\gamma} \left(\delta^{j(\omega,t)}/Q_t^{\phi} A_E\right)^{\gamma} + v_{LN}(j)$$

$$v_{LE}(j) = \frac{1}{\gamma} I_E^{\epsilon} \frac{\delta^{j(\omega,t)}}{Q_t^{\phi} A_E} + \frac{\delta^{j(\omega,t)}}{Q_t^{\phi} \delta A_F}$$

$$v_{LE}(j) = \frac{\delta^{j(\omega,t)}}{Q_t^{\phi}} \left(\frac{I_E^{\epsilon}}{\gamma A_E} + \frac{1}{\delta A_F}\right). \tag{5}$$

Finding the Labor Equation

Total production employment $L_P(t)$ can be expressed as:

$$\begin{aligned}
L_P(t) &= \int_{m_{LE}+m_{LN}} d(\omega,t) L_t d\omega + \tau \int_{m_{LE}} d(\omega,t) L_t d\omega + \int_{m_{CF}} d(\omega,t) L_t d\omega \\
&= \int_{m_{LE}+m_{LN}} \frac{q(\omega,t)}{Q_t} y(t) L_t d\omega + \tau \int_{m_{LE}} \frac{q(\omega,t)}{Q_t} y(t) L_t d\omega + \int_{m_{CF}} \frac{q(\omega,t)}{Q_t} y(t) \lambda^{\sigma} L_t d\omega \\
&= \frac{Q_{LE}+Q_{LN}}{Q_t} y(t) L_t + \tau \frac{Q_{LE}}{Q_t} y(t) L_t + \frac{Q_{CF}}{Q_t} y(t) \lambda^{\sigma} L_t \\
&= (q_{LE}+q_{LN})\, y(t) L_t + \tau q_{LE} y(t) L_t + q_{CF} y(t) \lambda^{\sigma} L_t \\
&= (q_{LE}+q_{LN}+\tau q_{LE}+\lambda^{\sigma} q_{CF})\, y(t) L_t.
\end{aligned}$$

Total R&D employment $L_R(t)$ can be expressed as:

$$\begin{aligned}
L_R(t) &= \int_{m_{LE}+m_{LN}} l\, d\omega + \int_{m_{LN}} l_E\, d\omega \\
&= \int_{m_{LE}+m_{LN}} \frac{q(\omega,t) Q_t^{-\phi} I}{A_F}\, d\omega + \int_{m_{LN}} \frac{q(\omega,t) Q_t^{-\phi} I_E^{1/\gamma}}{A_E}\, d\omega \\
&= \frac{Q_{LE}+Q_{LN}}{Q_t} \frac{Q_t^{1-\phi}}{L_t} \frac{I}{A_F} L_t + \frac{Q_{LN}}{Q_t} \frac{Q_t^{1-\phi}}{L_t} \frac{I_E^{1/\gamma}}{A_E} L_t \\
&= (q_{LE}+q_{LN}) x(t) \frac{I}{A_F} L_t + q_{LN} x(t) \frac{I_E^{1/\gamma}}{A_E} L_t \\
&= \left((q_{LE}+q_{LN}) I/A_F \;+\; q_{LN} I_E^{1/\gamma}/A_E\right) x(t) L_t.
\end{aligned}$$

Finding the R&D Equation

To find the R&D equation, we use the Bellman equation for a non-exporting leader: $rv_{LN}(j) = \pi_L(j) - l_E - Iv_{LN}(j) + I_E\left(v_{LE}(j) - v_{LN}(j)\right) + \dot{v}_{LN}(j)$. Using firm profits and substituting for l_E using $I_E = \left(Q_t^{\phi} A_E l_E / \delta^{j_{\omega,t}}\right)^{\gamma}$ yields

$$\begin{aligned} rv_{LN}(j) &= (\lambda-1)\delta^{j(\omega,t)}\frac{y(t)}{Q_t}L_t - I_E^{1/\gamma}\frac{\delta^{j(\omega,t)}}{Q_t^{\phi}A_E} \\ &\quad -Iv_{LN}(j) + I_E\left(v_{LE}(j) - v_{LN}(j)\right) + \dot{v}_{LN}(j). \end{aligned}$$

Now $v_{LN}(j) = \delta^{j(\omega,t)}/(Q_t^{\phi}\delta A_F)$ implies that $\dot{v}_{LN}(j)/v_{LN}(j) = -\phi\dot{Q}_t/Q_t$ during an R&D race and (5) implies that $v_{LE}(j) - v_{LN}(j) = \delta^{j(\omega,t)}I_E^{\epsilon}/(Q_t^{\phi}\gamma A_E)$. Thus, dividing the Bellman equation by $v_{LN}(j)$ and rearranging terms yields

$$r + I + \phi\frac{\dot{Q}_t}{Q_t} = (\lambda-1)\frac{\delta^{j(\omega,t)}}{v_{LN}(j)}\frac{y(t)}{Q_t}L_t - \frac{I_E^{1/\gamma}}{v_{LN}(j)}\frac{\delta^{j(\omega,t)}}{Q_t^{\phi}A_E} + \frac{I_E}{v_{LN}(j)}\frac{\delta^{j(\omega,t)}I_E^{\epsilon}}{Q_t^{\phi}\gamma A_E}.$$

Finally, after substituting for $v_{LN}(j)$ and simplifying

$$\begin{aligned} r + I + \phi\frac{\dot{Q}_t}{Q_t} &= (\lambda-1)\frac{\delta^{j(\omega,t)}}{\frac{\delta^{j(\omega,t)}}{Q_t^{\phi}\delta A_F}}\frac{y(t)}{Q_t}L_t - \frac{I_E^{1/\gamma}}{\frac{\delta^{j(\omega,t)}}{Q_t^{\phi}\delta A_F}}\frac{\delta^{j(\omega,t)}}{Q_t^{\phi}A_E} + \frac{I_E}{\frac{\delta^{j(\omega,t)}}{Q_t^{\phi}\delta A_F}}\frac{\delta^{j(\omega,t)}I_E^{\epsilon}}{Q_t^{\phi}\gamma A_E} \\ &= (\lambda-1)\delta A_F\frac{y(t)}{Q_t^{1-\phi}}L_t - \frac{\delta A_F}{A_E}I_E^{1/\gamma} + \frac{\delta A_F}{\gamma A_E}I_E^{\frac{\gamma}{\gamma}+\frac{1-\gamma}{\gamma}} \\ &= (\lambda-1)\delta A_F\frac{y(t)}{x(t)} + \frac{\delta A_F}{A_E}I_E^{1/\gamma}\frac{1-\gamma}{\gamma} \end{aligned}$$

we obtain the R&D equation

$$r + I + \phi\frac{\dot{Q}_t}{Q_t} = (\lambda-1)\delta A_F\frac{y}{x} + \frac{\delta A_F}{A_E}I_E^{1/\gamma}\epsilon. \tag{9}$$

Quality Dynamics

The dynamics of $Q_{LE} \equiv \int_{m_{LE}} \delta^{j(\omega,t)}d\omega$ is given by the differential equation

$$\begin{aligned} \dot{Q}_{LE} &= \int_{m_{LN}} \delta^{j(\omega,t)}I_E d\omega - \int_{m_{LE}} \delta^{j(\omega,t)}I d\omega \\ &= Q_{LN}I_E - Q_{LE}I \end{aligned}$$

and dividing by Q_{LE} yields

$$\begin{aligned}\frac{\dot{Q}_{LE}}{Q_{LE}} &= \frac{Q_{LN}}{Q_{LE}} I_E - I \\ &= \frac{q_{LN}}{q_{LE}} I_E - I.\end{aligned}$$

The dynamics of $Q_{LN} \equiv \int_{m_{LN}} \delta^{j(\omega,t)} d\omega$ is given by the differential equation

$$\begin{aligned}\dot{Q}_{LN} &= \int_{m_{LN}} \left(\delta^{j(\omega,t)+1} - \delta^{j(\omega,t)}\right) I d\omega - \int_{m_{LN}} \delta^{j(\omega,t)} I_E d\omega + \int_{m_{LE}} \delta^{j(\omega,t)+1} I d\omega \\ &= (\lambda - 1) Q_{LN} I - Q_{LN} I_E + \delta Q_{LE} I\end{aligned}$$

and dividing by Q_{LN} yields

$$\begin{aligned}\frac{\dot{Q}_{LN}}{Q_{LN}} &= (\delta - 1) I - I_E + \delta \frac{Q_{LE}}{Q_{LN}} I \\ &= (\delta - 1) I - I_E + \delta \frac{q_{LE}}{q_{LN}} I.\end{aligned}$$

The dynamics of $Q_{CF} \equiv \int_{m_{CF}} \delta^{j(\omega,t)} d\omega$ is given by the differential equation

$$\begin{aligned}\dot{Q}_{CF} &= \int_{m_{LE}} \delta^{j(\omega,t)} I d\omega - \int_{m_{CF}} \delta^{j(\omega,t)} I_E d\omega \\ &= Q_{LE} I - Q_{CF} I_E\end{aligned}$$

and dividing by Q_{CF} yields

$$\begin{aligned}\frac{\dot{Q}_{CF}}{Q_{CF}} &= \frac{Q_{LE}}{Q_{CF}} I - I_E \\ &= \frac{q_{LE}}{q_{CF}} I - I_E.\end{aligned}$$

Now using (10), we obtain that $\dot{Q}_{LE}/Q_{LE} = (q_{LN}/q_{LE}) I_E - I = n/(1-\phi)$ and solving for I yields

$$I = \frac{q_{LN}}{q_{LE}} I_E - \frac{n}{1-\phi}.$$

Using (10) again, we obtain that $\dot{Q}_{LN}/Q_{LN} = (\delta - 1) I - I_E + \delta (q_{LE}/q_{LN}) I = n/(1-\phi)$ and solving for I yields

$$I = \frac{I_E + \frac{n}{1-\phi}}{\delta - 1 + \delta \frac{q_{LE}}{q_{LN}}}.$$

It immediately follows that

$$\frac{I_E + \frac{n}{1-\phi}}{\delta - 1 + \delta \frac{q_{LE}}{q_{LN}}} = \frac{q_{LN}}{q_{LE}} I_E - \frac{n}{1-\phi}$$

and rearranging terms using the new variable $z \equiv q_{LN}/q_{LE}$ yields

$$\begin{aligned} I_E + \frac{n}{1-\phi} &= \left(z I_E - \frac{n}{1-\phi} \right) \left(\delta - 1 + \delta z^{-1} \right) \\ I_E &= z I_E (\delta - 1) + I_E \delta - \frac{n}{1-\phi}\delta - \frac{n}{1-\phi}\delta z^{-1} \\ 0 &= z I_E (\delta - 1) + \left(I_E(\delta - 1) - \frac{n}{1-\phi}\delta \right) - \frac{n}{1-\phi}\delta z^{-1}. \end{aligned}$$

Multiplying both sides of the last equation by z then yields a quadratic equation in z

$$0 = z^2 I_E (\delta - 1) + z \left(I_E(\delta - 1) - \frac{n}{1-\phi}\delta \right) - \frac{n}{1-\phi}\delta.$$

and solving this equations using the quadratic formula, we obtain two solutions

$$z_{1,2} = \frac{-\left(I_E(\delta - 1) - \frac{n}{1-\phi}\delta \right) \pm \left(\left(I_E(\delta - 1) - \frac{n}{1-\phi}\delta \right)^2 + 4 I_E (\delta - 1) \frac{n}{1-\phi}\delta \right)^{1/2}}{2 I_E (\delta - 1)}.$$

Expanding the expression under the square root, we obtain

$$\begin{aligned} & \left(I_E(\delta - 1) - \frac{n}{1-\phi}\delta \right)^2 + 4 I_E (\delta - 1) \frac{n}{1-\phi}\delta \\ = \; & \left(I_E(\delta - 1) \right)^2 + \left(\frac{n}{1-\phi}\delta \right)^2 + 2 I_E (\delta - 1) \frac{n}{1-\phi}\delta \\ = \; & \left(I_E(\delta - 1) + \frac{n}{1-\phi}\delta \right)^2. \end{aligned}$$

It follows that the two solutions to the quadratic equation are

$$z_{1,2} = \frac{\left(-I_E(\delta - 1) + \frac{n}{1-\phi}\delta \right) \pm \left(I_E(\delta - 1) + \frac{n}{1-\phi}\delta \right)}{2 I_E (\delta - 1)}$$

and since z must be positive to be economically meaningful, I can focus on the positive solution

$$z \equiv \frac{q_{LN}}{q_{LE}} = \frac{n\delta}{(1-\phi) I_E (\delta - 1)}.$$

Plugging this solution back into $\dot{Q}_{LE}/Q_{LE} = (q_{LN}/q_{LE}) I_E - I = n/(1-\phi)$, we can uniquely

determine the steady-state equilibrium innovation rate I:

$$\begin{aligned} I &= \frac{q_{LN}}{q_{LE}} I_E - \frac{n}{1-\phi} \\ &= \frac{n\delta}{(1-\phi) I_E (\delta - 1)} I_E - \frac{n}{1-\phi} \\ &= \frac{n}{(1-\phi)} \frac{\delta - (\delta - 1)}{(\delta - 1)} \\ &= \frac{n}{(1-\phi)(\delta - 1)}. \end{aligned}$$

Plugging this result back into our solution for z, we obtain

$$\begin{aligned} q_{LN} &= q_{LE} \frac{n\delta}{(1-\phi) I_E (\delta - 1)} \\ &= q_{LE} \frac{I\delta}{I_E}. \end{aligned}$$

Using (10) one more time, we obtain that $\dot{Q}_{CF}/Q_{CF} = (q_{LE}/q_{CF})I - I_E = n/(1-\phi)$ and solving for I yields

$$I = \frac{q_{CF}}{q_{LE}} \left(\frac{n}{1-\phi} + I_E \right).$$

Combining this with the earlier result $I = (q_{LN}/q_{LE})I_E - n/(1-\phi)$ and rearranging terms, we obtain

$$\begin{aligned} \frac{q_{CF}}{q_{LE}} \left(\frac{n}{1-\phi} + I_E \right) &= \frac{q_{LN}}{q_{LE}} I_E - \frac{n}{1-\phi} \\ \frac{n}{1-\phi} + I_E &= \frac{q_{LN}}{q_{CF}} I_E - \frac{q_{LE}}{q_{CF}} \frac{n}{1-\phi} \\ I(\delta - 1) + I_E &= \frac{q_{LE}}{q_{CF}} \frac{I\delta}{I_E} I_E - \frac{q_{LE}}{q_{CF}} I(\delta - 1) \\ I(\delta - 1) + I_E &= \frac{q_{LE}}{q_{CF}} I \\ q_{CF} &= q_{LE} \frac{I}{I(\delta - 1) + I_E}. \end{aligned}$$

It remains to determine the quality share q_{LE}. Substituting into (7) and rearranging

terms, we obtain

$$\begin{aligned}
1 &= 2q_{LE} + q_{LN} + q_{CF} \\
&= 2q_{LE} + q_{LE}\frac{I\delta}{I_E} + q_{LE}\frac{I}{I(\delta-1)+I_E} \\
&= q_{LE}\left(2 + \frac{I\delta}{I_E} + \frac{I}{I(\delta-1)+I_E}\right) \\
q_{LE} &= \left(2 + \frac{I\delta}{I_E} + \frac{I}{I(\delta-1)+I_E}\right)^{-1}.
\end{aligned}$$

Finding I_E

The Bellman equation for an exporting leader is $rv_{LE}(j) = \pi_L(j) + \pi_E(j) - Iv_{LE}(j) + \dot{v}_{LE}(j)$. Substituting into this Bellman equation for $\pi_L(j)$ and $\pi_E(j)$ yields

$$rv_{LE}(j) = (\lambda-1)\frac{\delta^{j(\omega,t)}}{Q_t}yL_t + (\lambda-\tau)\frac{\delta^{j(\omega,t)}}{Q_t}yL_t - Iv_{LE}(j) + \dot{v}_{LE}(j).$$

Next, we divide both sides of this equation by $v_{LE}(j)$ and substitute for $v_{LE}(j)$ using (5). Taking into account that $\dot{v}_{LE}/v_{LE} = -\phi\dot{Q}_t/Q_t$ follows from (5) in any steady-state equilibrium where I_E is constant over time, we obtain

$$\begin{aligned}
r &= \frac{2\lambda-1-\tau}{v_{LE}(j)}\frac{\delta^{j(\omega,t)}}{Q_t}yL_t - I + \frac{\dot{v}_{LE}(j)}{v_{LE}(j)} \\
r &= \frac{2\lambda-1-\tau}{I_E^{\epsilon}/(\gamma A_E) + 1/\delta A_F}\frac{Q_t^{\phi}\delta^{j(\omega,t)}}{Q_t\delta^{j(\omega,t)}}yL_t - I - \phi\dot{Q}_t/Q_t \\
r + I + \phi\dot{Q}_t/Q_t &= \frac{2\lambda-1-\tau}{I_E^{\epsilon}/(\gamma A_E) + 1/\delta A_F}\frac{L_t}{Q_t^{1-\phi}}y \\
r + I + \phi\dot{Q}_t/Q_t &= \frac{2\lambda-1-\tau}{I_E^{\epsilon}/(\gamma A_E) + 1/\delta A_F}\frac{y}{x}.
\end{aligned}$$

Solving the above expression for y/x and then substituting into the R&D equation (9), we obtain

$$r + I + \phi\dot{Q}_t/Q_t = (\lambda-1)\delta A_F\frac{\left(r + I + \phi\dot{Q}_t/Q_t\right)\left(I_E^{\epsilon}\frac{1}{\gamma A_E} + \frac{1}{\delta A_F}\right)}{2\lambda-1-\tau} + \frac{\delta A_F}{A_E}I_E^{\frac{1}{\gamma}}\epsilon.$$

Then dividing both sides of this equation by $\left(r + I + \phi\dot{Q}_t/Q_t\right)\delta A_F$ yields

$$\frac{1}{\delta A_F} = \frac{\lambda-1}{2\lambda-1-\tau}\left(I_E^{\epsilon}\frac{1}{\gamma A_E} + \frac{1}{\delta A_F}\right) + \frac{I_E^{\frac{1}{\gamma}}\epsilon}{A_E\left(r + I + \phi\dot{Q}_t/Q_t\right)}. \tag{11}$$

The Steady-State Equilibrium

In steady-state equilibrium, consumer utility at time t is given by

$$\begin{aligned} u_t &= \frac{c_t}{P_t} \\ &= \frac{y\lambda^\sigma P_t^{1-\sigma}}{Q_t P_t} \\ &= \frac{y\lambda^\sigma}{Q_t}\left(P_t^{1-\sigma}\right)^{\sigma/(\sigma-1)} \\ &= \frac{y\lambda^\sigma}{Q_t^{(\sigma-1)/(\sigma-1)}}\left[(2q_{LE}\lambda^{1-\sigma} + q_{LN}\lambda^{1-\sigma} + q_{CF})Q_t\right]^{\sigma/(\sigma-1)}, \end{aligned}$$

from which it follows that

$$u_t = y\lambda^\sigma Q_t^{\frac{1}{\sigma-1}}\left[(2q_{LE} + q_{LN})\lambda^{1-\sigma} + q_{CF}\right]^{\frac{\sigma}{\sigma-1}}. \tag{12}$$

Taking logs of both sides and then differentiating with respect to t yields the steady-state rate of economic growth

$$g_u \equiv \frac{\dot{u}_t}{u_t} = \frac{1}{\sigma-1}\frac{\dot{Q}_t}{Q_t} = \frac{n}{(\sigma-1)(1-\phi)}.$$

Firm Exit

Solving for the firm exit rate or death rate N_D yields

$$\begin{aligned} N_D &\equiv \frac{Im_{LN} + Im_{LE} + (I_E + I)2m_{CF}}{m_{LN} + m_{LE} + 2m_{CF}} \\ &= \frac{I\frac{I/2}{I+I_E} + I\frac{I_E/2}{I+I_E} + (I_E + I)2\frac{I/2}{I+I_E}}{\frac{I/2}{I+I_E} + \frac{I_E/2}{I+I_E} + 2\frac{I/2}{I+I_E}} \\ &= \frac{I(I + I_E + 2I_E + 2I)}{3I + I_E} \\ &= \frac{3I(I + I_E)}{3I + I_E}. \end{aligned}$$

It follows that

$$\frac{\partial N_D}{\partial I_E} = \frac{(3I + I_E)3I - 3I(I + I_E)}{(3I + I_E)^2} = \frac{6I^2}{(3I + I_E)^2} > 0.$$

Comparing Exporters and Non-Exporters

Under what conditions is the average quality of products produced by exporters higher than the average quality of products produced by non-exporters, or $Q_E > Q_N$? Exploring when this inequality holds, we obtain

$$\begin{aligned}
\frac{q_{LE}}{m_{LE}}Q_t &> \frac{q_{LN}+2q_{CF}}{3m_{LN}}Q_t \\
\frac{3m_{LN}}{m_{LE}} &> \frac{q_{LN}+2q_{CF}}{q_{LE}} \\
\frac{3\frac{I/2}{I+I_E}}{\frac{I_E/2}{I+I_E}} &> \frac{q_{LE}\frac{I\delta}{I_E}+2q_{LE}\frac{I}{I(\delta-1)+I_E}}{q_{LE}} \\
\frac{3I}{I_E} &> \frac{I\delta}{I_E}+\frac{2I}{I(\delta-1)+I_E} \\
3 &> \delta+\frac{2I_E}{I(\delta-1)+I_E}.
\end{aligned}$$

Assuming that $3 > \delta$, $Q_E > Q_N$ holds when

$$\begin{aligned}
(3-\delta)I(\delta-1)+(3-\delta)I_E &> 2I_E \\
(3-\delta)I(\delta-1) &> (\delta-1)I_E \\
(3-\delta)I &> I_E.
\end{aligned}$$